The FWM Impairment in Coherent OFDM Compounds on a Phased-Array Basis over Dispersive Multi-Span Links

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Abstract: We develop a novel all-analytic model of FWM generation over dispersive multi-span coherent OFDM long-haul links, leading to a new phased-array effect. The nonlinear FWM impairment may be mitigated by destructive interference of intermodulation products.

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1. Introduction and overview

Coherent Optical Orthogonal Frequency Division Multiplexing (CO-OFDM) is emerging as a key optical transmission technique [1-4], resilient to *Chromatic Dispersion* (CD) and PMD, due to the long symbol length of the individual data tributaries carried over a multitude of low-rate orthogonal sub-channels. The residual CD may be further suppressed by digital *Dispersion Compensation* (DC) in the frequency domain. The remaining main factor limiting performance is then the *Four-Wave-Mixing* (FWM) impairment. Lowery [2] provided an analytic estimate of FWM generation, assuming a *dispersion-free* system. We point out that although the dispersive phase mismatch walkoff between adjacent subcarriers is negligible, the overall effect of CD on FWM over the full frequency band may be appreciable for a high-speed OFDM system occupying an extended bandwidth of tens of GHz (the dispersive walkoff is large between carriers well separated in the band). It follows that by neglecting CD, [2] merely provided a loose upper bound on the impact of the FWM impairment. An understanding of the interaction between CD and FWM, including possible interactions among multiple spans, is critical to CO-OFDM system analysis and design.

This paper develops for the first time a rigorous *analytic model of BER performance* of long-haul CO-OFDM systems, evaluating the Q-factors for QPSK transmission over the sub-carriers in terms of compact closed-form expressions averaging the contributions of all triplets of *intermodulation products* (IPs), properly accounting for the reduction in the efficiency of FWM generation due to CD-induced phase-mismatch. A new consequence of our analysis is that, unlike predicted in [2] for the CD-free case, the FWM contributions of the individual spans in a multi-span long-haul link do *not* add up in-phase. Rather, we prove that *the individual fiber spans act as antennas in a very long one-dimensional phased-array*. Akin to the presence of nulls in the radiation pattern of a phased-array, destructive interference may onset between the FWM contributions of the individual fiber spans, e.g. for the OFDM system described in this paper, we attain ~16.5 dB of FWM reduction, averaged over all IPs.

We developed our analytical model to account for CD+FWM. The model was then applied to analyze the performance of a state-of-the-art coherent system transmitting a 102.4 *Gbps* OFDM signal comprising *M*=128 QPSK sub-channels, each operating at 200 *Msym/sec* with polarization multiplexing: $128Ch \cdot 200\frac{Msym/\sec\cdot}{Ch\cdot Pol}\cdot 2\frac{bit}{sym}\cdot 2\,pol = 102.4\frac{Gbit}{\sec}$. At the subcarrier separation of $\Delta\nu = 200$ MHz, the total OFDM *Bandwidth* is $B = M\Delta\nu = 25.6$ GHz. Previewing our key findings, we first assume that the only mechanism present were the reduction in FWM efficiency over each individual span due to phase-mismatch. Then such hypothesis would yield a $BER = 10^{-4}$ over 30 spans, each of $L_{span} = 80\,Km$ (assume G.652 standard fiber with $D = 17\frac{p\sec}{nm\cdot Km}$, $\alpha = 0.22\,dB/Km$, and optical amplifier gain and noise figure $G = e^{\alpha L_{span}}$, $F_N = 6.5\,dB$). Remarkably, once our newly discovered *phased-array FWM effect* is modeled in, the ideal transmission range can be significantly extended from 30 spans (2400 Km) to 82 spans (6560 Km).

2. Analysis of dispersive FWM including the phased-array multi-span effect

Our analytic model is rigorous within the framework of the *undepleted pumps assumption* [5] (which is very accurate given the relatively low level of *NonLinear* (NL) FWM fluctuations required at $BER = 10^{-4}$). Here the "pumps" are the QPSK-modulated subcarriers. We worked out the FWM generation by two alternative independent methods:

(i): A differential method, starting from the *NonLinear Schroedinger Equation* [5], accounting for the CD effect upon the NL driving term in the RHS of the equation, but neglecting the CD in the LSH, since each of the subcarrier waveforms is very slow. (ii): A path integral method, partitioning the multi-span link into a multitude of differential length elements, propagating all triplets of pumps from the input to each element dz at z, working out the IP induced in each element, propagating the IP secondary field to the output, and superposing the contribution of all elements by integrating over z. Both methods yield the same analytic expressions, enabling OFDM systems analysis and design: $BER[i] = 2Q\left[\left(q_{FWM}^{-2}[i] + q_{LN}^{-2}[i]\right)^{-1/2}\right]$ where $q_{\angle LN}$ is the Q-factor of ASE-induced linear phase noise, evaluated as in [4], the FWM-related Q-factor is $q_{FWM}[i] = (\pi/4)/\left[\left\langle F_{ijk}\hat{L}_{ijk}^{FWM}\right\rangle_{rms}\sigma_{\angle FWM}^{\text{noCD}}\right]$, with $\left(\sigma_{\angle FWM}^{\text{noCD}}\right)^2$ the power of FWM-induced phase fluctuations, evaluated in [2] in the absence of dispersion. Here $j, k, l \equiv j + k - i$ are indexes of subchannel triplets with IPs falling on the i-th subchannel, forming the FWM IPs set (excluding XPM and SPM): $S_i \equiv \{[j,k] \mid 1 \le j,k \le M \ \&\ 0 < j+k-i \le M \ \&\ j \ne i \ne k\}$, and $\langle\cdot\rangle_{rms}$ denotes root-mean-square averaging (over all pairs of this set) for two new parameters: $\hat{L}_{ijk}^{FWM} = L_{eff}^{-1}(1 - e^{-\alpha L}e^{-j\Delta\beta_{ijk}L})/(j\Delta\beta_{ijk} + \alpha)$ describing the *per-span mismatch* (PSM) of FWM due to the four-wave $\Delta\beta$ -phase detuning $\Delta\beta_{ijk} = -\beta''(2\pi\Delta\nu)^2(j-i)(k-i)$, and a *phased-array factor* (PAF) $F_{ijk}[N_{span}] = \sin(\pi u)/\left(N_{span}\sin(\pi u/N_{span})\right)$ *with* $u = N_{span}L_{span}\Delta\beta_{ijk}/2\pi$ describing multi-span FWM interference between the spans, beyond their mere juxtaposition into a long link. The PAF-induced improvement is intuitively illustrated in Fig. 1, describing the phasor addition of the regularly dephased FWM contributions from each of the spans, forming a polygon tending to close onto itself, yielding a reduced or even null resultant.

As PSM and PAF are both <1, the Q-factor is improved by the dispersion, however (for the system considered) the PSM provides just ~1 dB improvement (over the whole link) relative to the dispersion-free case, whereas the rms PAF reduces the overall FWM power by ~16.5 dB! This substantial FWM cancellation is made plausible by verifying that the majority of IP triplets experience sufficient dispersive mismatch to place them in the sidelobes region of the PAF graph (Fig. 1b). For a given fiber (fixed CD parameter β''), higher dispersion may be obtained by increasing the $\Delta\nu$ intercarrier separation (higher $\Delta\beta_{ijk}$), implying fewer carriers, at a higher rate per carrier in a given BW. Based on these analytic expressions, we simulated the 100 Gbps OFDM system targeted in the intro, optimizing the distance and optical power for worst-case $BER = 10^{-4}$. In Fig. 2 we merely model the per-span FWM reduction, whereas Fig. 3 realistically models both per-span mismatch and PAF, demonstrating a dramatic increase in transmission range attained due to the destructive interference effect of the phased-array FWM compounding over multiple spans.

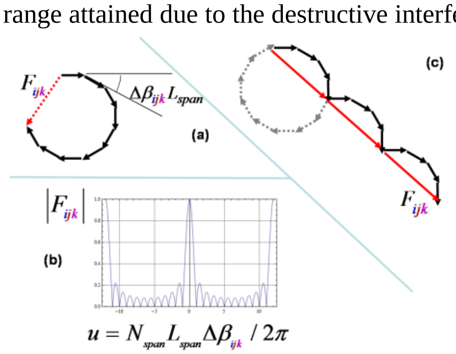

Fig. 1: Phasor addition of the FWM contributions of the multiple spans (a): CD uncompensated link (b): Array Factor plot. (c): Link with periodic compensation every few spans. Here the phased-array amplitude reduction is less favorable.

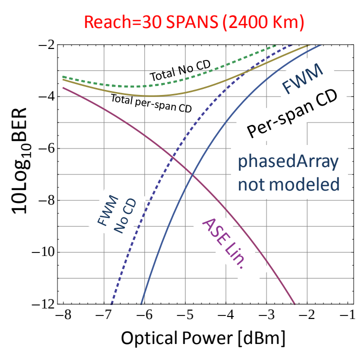

Fig. 2: BER performance for CD uncompensated link of **30** spans. Top plot is the total BER (10^{-4} at -5.5 dBm), ignoring the phased-array effect, while accounting for dispersive mismatch in each span.

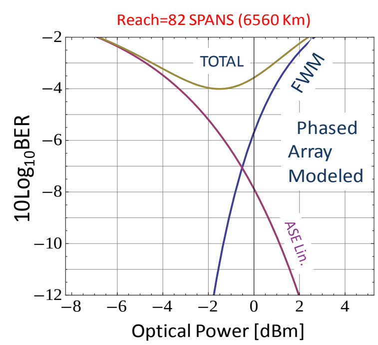

Fig. 3: BER performance for CD uncompensated link of **82** spans. Top plot is total BER (10^{-4} at -1.5 dBm). The substantial increase in range vs. Fig. 1 stems from the phased-array cancellation effect of the FWM impairment.

To conclude, OFDM may work much better than previously thought, under certain configurations. The newly discovered beneficial phased-array FWM reduction effect favors OFDM systems with higher dispersion, i.e. higher subcarrier separation $\Delta\nu \ge 200MHz$, as in next generation higher-speed links (e.g. 100 Gbps) using non-dispersion shifted fiber, and having the in-line DCF modules not installed (or removed), relying on OFDM DC at the link ends.

6. References

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