Optical Phase-Locked Loop Performance in Homodyne Detection using Pulsed and CW LO

P. S. Cho
CeLight, Inc., 12200 Tech Road, Suite 300, Silver Spring, Maryland 20904
E-mail: pscho@celight.com

Abstract: Theoretical comparison of balanced and Costas optical phase-locked loops performance in homodyne detection using pulsed- and CW-LO is reported. Analytical expressions of the total phase error variances for pulsed-LO in terms of CW-LO are presented.

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1. Introduction

Conventional homodyne detection employs a continuous-wave (CW) local laser oscillator (LO) to achieve down-conversion of the optical signal to the base-band and amplification simultaneously. Recent experiment on homodyne detection of 12.5 Gb/s binary phase-shift-keyed (BPSK) signals using pulsed LO showed that the receiver sensitivity is at least 2 dB better than that for CW LO [1]. Pulsed LO also offers pulse shaping capabilities [1] and the potential of optical demultiplexing of OTDM signal not shared by CW LO. The performance of optical phase-locked loop (OPLL) using pulsed LO, however, has not been reported to the best knowledge of the author. In this paper, theoretical comparison of the performance of balanced and Costas OPLLs using pulsed and CW LO is described. Analytical expressions of the total phase error variances for the two types of OPLLs as a result of laser phase noise, data noise, and shot noise for pulsed LO in terms of those for the CW LO are presented.

In this analysis, the pulsed LO is produced by a push-pull type Mach-Zehnder modulator (MZM) with an output optical field at a carrier frequency f_c: $E_{LO}^{P}(t) = E_i \sin\left[\pi V(t)/(2V_\pi)\right] e^{j2\pi f_c t}$, where V_π is the MZM's half-wave voltage and the driving voltage of the MZM is $V(t) = V_\pi\left[k_b - k_s \sin(k_d \omega_m t)\right]/2$. The RF modulation frequency, $\omega_m/(2\pi)$, equals to the symbol rate ($1/T_s$) of the optical signal to be mixed with the LO. Fig. 1a shows values of k_b, k_s, and k_d that produce pulses with duty cycles of 50, 35, and 65%. The 65% duty cycle case is not considered here since it is not suitable for the OPLL due to carrier-suppressed pulses. The duty cycle is defined here as the ratio of the average power to the peak power of the pulse. For fair comparison, the average power of the pulsed LO is assumed to be the same as the CW LO (P_{LO}^{CW}). Therefore, the pulsed LO power, proportional to $\left|E_{LO}^{P}(t)\right|^2$, can be expanded as follows:

$$P_{LO}^{P}(t) = \left(P_{LO}^{CW}/D\right)\left\{D - \cos(\pi k_b/2)\sum_{\substack{n=2\\ even}}^{\infty}\left[J_n(\pi k_s/2)\cos(n k_d \omega_m t)\right] - \sin(\pi k_b/2)\sum_{\substack{n=1\\ odd}}^{\infty}\left[J_n(\pi k_s/2)\sin(n k_d \omega_m t)\right]\right\}, \quad (1)$$

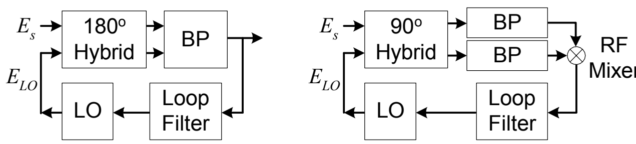
and $$\sqrt{P_{LO}^{P}(t)} = \sqrt{\left(P_{LO}^{CW}/D\right)}\left\{\sin(\pi k_b/4) J_0(\pi k_s/4) + \text{summation of terms of } \sin(n k_d \omega_m t) \text{ and } \cos(n k_d \omega_m t)\right\}, \quad (2)$$

where $D = \left[1 - \cos(\pi k_b/2) J_0(\pi k_s/2)\right]/2$ is the duty cycle and $J_n(\bullet)$ is the n^{th} order Bessel function of the first kind.

Balanced and Costas loops are two well-known OPLLs. Many theoretical analyses on their performances with CW LO have been reported previously [2,3]. Similar analysis can be applied for pulsed LO taking into account of its time-varying response. Figs. 1b and 1c show schematics of the two OPLLs. The performance of an OPLL can be quantified by the phase error ϕ_e and its variance $\sigma_{\phi_e}^2$ with $\phi_e(t) = \phi_{Tx}(t) - \phi_{LO}(t) - \phi_c(t)$, where ϕ_{Tx} and ϕ_{LO} are the transmitter and local laser phase noise, and ϕ_c is the controlled phase to the LO. For balanced OPLL, the total phase error variance is $\sigma_{\phi_e}^2 = \sigma_l^2 + \sigma_s^2 + \sigma_d^2$ where the terms on the right side represents independent noise processes: signal and LO laser phase noise (σ_l^2), shot noise (σ_s^2), and data-induced noise (σ_d^2) [2]. For Costas OPLL, the total phase error variance is $\sigma_{\phi_e}^2 = \sigma_l^2 + \sigma_s^2$ with no contribution from data noise [3]. Detail derivation of $\sigma_{\phi_e}^2$ for the two OPLLs with pulsed LO is quite lengthy therefore only results are presented here. The analysis assumed that the phase error is small so that the OPLL is tracking the phase which means $\sin[\phi_e(t)] \approx \phi_e(t)$ and $\cos[\phi_e(t)] \approx 1$. Also, the OPLL has a first-order low-pass loop filter (Figs. 1b and 1c) with a bandwidth much smaller than the symbol rate of the signal which is typically the case [2]. The data signal is assumed to be non-return-to-zero (NRZ) format. For fair comparison, parameters such as the loop filter time constants (τ_1 and τ_2) and the loop gain (G_L) are assumed to be the same for both CW and pulsed LO [2]. Equations (1) and (2) were used to derive the noise power spectral densities for the phase error variances for pulsed LO in terms of those for CW LO. Except for the laser phase noise induced phase error, the phase error variance can be evaluated via the integral: $I_i = \int_{-\infty}^{\infty} G_i \left|H_{PLL}(\omega)\right|^2 d\omega$, where $H_{PLL}(\omega)$ depends on the transfer function of the loop filter, $H_{LF}(\omega)$, and G_i is the power spectral density of the various noise processes [2]. For balanced OPLL, G_i is proportional to $\left|F\left\{P_{LO}^{P}(t)\,d(t)\right\}\right|^2$ and $\left|F\left\{n_s(t)\right\}\right|^2$ for data and shot noise induced phase error, where $F\{\bullet\}$ denotes Fourier transform, $d(t)$ varies between ±1 represents the binary NRZ data, and $n_s(t)$ is the shot noise. For Costas OPLL, G_i is proportional to $\left|F\left\{\sqrt{P_{LO}^{P}(t)}\,d(t)\,n_s(t)\right\}\right|^2$ for shot noise induced phase error. Using Fourier transform properties and the limited bandwidth of the loop filter, it can be shown that all the $\sin(nk_d\omega_m t)$ and $\cos(nk_d\omega_m t)$ terms in Eq. (1) and Eq. (2) do not contribute to the integral I_i and can be dropped. As a result, the phase error analysis can be significantly simplified resulting into analytical expressions of the phase error variances for pulsed LO for balanced and Costas OPLLs. These results are presented next.

Duty Cycle	k_b	k_s	k_d
50%	1	1	1
35%	2	2	1/2
65%	0	2	1/2

(a)



(b) Balanced Loop (c) Costas Loop

Fig. 1. (a) Parameters of pulsed LO. (b) Balanced and (c) Costas OPLLs. LO: CW or pulsed. BP: balanced photoreceiver.

2. Balanced OPLL

The phase error variance due to laser phase noise for a beat linewidth of $\Delta\nu$ using CW LO is given by [2]: $\sigma_{l-CW}^2 = 2\pi\Delta\nu \big/ \left(\xi^{CW}\omega_n^{CW}\right)$, where $\omega_n^{CW} = \sqrt{A_L^{CW}\,G_L/\tau_1}$, $\xi^{CW} = \omega_n^{CW}\tau_2$, $A_L^{CW} = 2C_g\sqrt{P_s P_{LO}^{CW}}\cos(\theta)$, C_g is the product of the transimpedance gain (G_T) and the photodetector responsivity (R), P_s and P_{LO}^{CW} are the average optical powers of the signal and the CW LO, $\theta < \pi/2$ is the phase angle of the incomplete modulation to produce the required residual pilot carrier (full modulation: $\theta = \pi/2$) [2]. The natural frequency and damping coefficient of the loop are $\omega_n^{CW}/(2\pi)$ and $\xi^{CW}/2$, respectively. For pulsed LO, the laser phase noise induced phase error variance is $\sigma_{l-P}^2 = 2\pi\Delta\nu \big/ \left(\xi^{P}\omega_n^{P}\right)$ with $\omega_n^{P} = \sqrt{A_L^{CW}\,G_L/\tau_1} \big/ \sqrt{B_l} = \omega_n^{CW} \big/ \sqrt{B_l}$ and $\xi^{P} = \omega_n^{P}\tau_2 = \omega_n^{CW}\tau_2 \big/ \sqrt{B_l} = \xi^{CW} \big/ \sqrt{B_l}$ where $B_l = \sqrt{D} \big/ \left[\sin(\pi k_b/4)\,J_0(\pi k_s/4)\right]$. Therefore, we obtain $\sigma_{l-P}^2 = 2\pi\Delta\nu \big/ \left(\xi^{P}\omega_n^{P}\right) = B_l\,2\pi\Delta\nu \big/ \left(\xi^{CW}\omega_n^{CW}\right) = B_l\sigma_{l-CW}^2$.

The phase error variance due to data noise for CW LO is given by [2]: $\sigma_{d-CW}^2 = 2T_s\tan^2(\theta)\,B_{PLL}^{CW}$, where $B_{PLL}^{CW} = \left[\omega_n^{CW} \big/ \left(4\xi^{CW}\right)\right]\left[1+\left(\xi^{CW}\right)^2\right]$. For pulsed LO, $\sigma_{d-P}^2 = 2T_s B_l^2\tan^2(\theta)\,B_{PLL}^{P} \big/ B_l^2 = 2T_s\tan^2(\theta)\,B_{PLL}^{P}$ is the data noise induced phase error variance where $B_{PLL}^{P} = \left[\omega_n^{P} \big/ \left(4\xi^{P}\right)\right]\left[1+\left(\xi^{P}\right)^2\right] = B_{PLL}^{CW}\left[1+\left(\xi^{CW}\right)^2 \big/ B_l\right] \Big/ \left[1+\left(\xi^{CW}\right)^2\right]$. Therefore, one obtain for the phase error variance due to data noise for pulsed LO:

$\sigma_{d-P}^2 = 2T_s\tan^2(\theta)\,B_{PLL}^{CW}\left[1+\left(\xi^{CW}\right)^2 \big/ B_l\right] \Big/ \left[1+\left(\xi^{CW}\right)^2\right] = B_d\sigma_{d-CW}^2$, where $B_d = \left[1+\left(\xi^{CW}\right)^2 \big/ B_l\right] \Big/ \left[1+\left(\xi^{CW}\right)^2\right]$.

The phase error variance due to shot noise for CW LO is given by [2]: $\sigma_{s-CW}^2 = \left(2qRP_{LO}^{CW}G_T^2\right)B_{PLL}^{CW} \big/ \left(A_L^{CW}\right)^2$. For pulsed LO, the phase error variance due to shot noise is $\sigma_{s-P}^2 = B_l^2 B_d\left(2qRP_{LO}^{CW}G_T^2\right)B_{PLL}^{CW} \big/ \left(A_L^{CW}\right)^2 = B_l^2 B_d\sigma_{s-CW}^2$. Therefore, the total phase error variance of balanced OPLL for pulsed LO in terms of that for CW LO is $\sigma_{\phi_e-P}^2 = B_l\sigma_{l-CW}^2 + B_d\sigma_{d-CW}^2 + B_l^2 B_d\sigma_{s-CW}^2$. Using pulsed LO parameters shown in Fig. 1a and assuming $\xi^{CW}/2 = 1/\sqrt{2}$ for typical damping coefficient, the total phase error variance of balanced OPLL for pulsed LO in terms of phase error variances for CW LO for 50 and 35% duty cycle of the LO pulse is

$$\sigma_{\phi_e-P}^2 = \begin{cases} 1.1742\sigma_{l-CW}^2 + 0.9011\sigma_{d-CW}^2 + 1.2424\sigma_{s-CW}^2, & \text{for 50\% duty cycle,} \\ 1.2496\sigma_{l-CW}^2 + 0.8668\sigma_{d-CW}^2 + 1.3536\sigma_{s-CW}^2, & \text{for 35\% duty cycle.} \end{cases}$$

The increase of the laser phase noise and shot noise induced phase error variances for pulsed LO is due to the reduced field amplitude of the optical carrier of the pulsed LO since its average power is the same as the CW LO. The data noise induced phase error is lower for pulsed LO due to reduction of the effective bandwidth of the OPLL caused by the reduced carrier field amplitude of the pulsed LO. Intuitively, the data noise effect should decrease since the un-intentional tracking of the low-frequency data-noise by the loop is less effective with a reduced optical carrier amplitude of the pulsed LO. To estimate the receiver sensitivity penalty due to the phase error for pulsed LO relative to CW LO, Gaussian distribution was used to approximate the phase error statistics in order to simplify the error probability calculation. The error probability is therefore given by [2]

$$P_e = \left[1\Big/\left(2\sqrt{2\pi}\sigma_{\phi_e}\right)\right]\int_{-\infty}^{\infty}\operatorname{erfc}\left[\sqrt{2N_R\sin^2(\theta+\phi_e)}\right]\exp\left[-\phi_e^2\Big/\left(2\sigma_{\phi_e}^2\right)\right]d\phi_e \,,$$

where N_R is the received number of photons per bit. Assuming that the shot noise induced phase error dominates all other noise process so that $\sigma_{\phi_e-CW}^2 \approx \sigma_{s-CW}^2 \Rightarrow \sigma_{\phi_e-P}^2 \approx B_l^2 B_d \sigma_{s-CW}^2 \Rightarrow \sigma_{\phi_e-P}^2 \approx B_l^2 B_d \sigma_{\phi_e-CW}^2$. Therefore, for $\sigma_{\phi_e-CW} = 5^\circ\ (10^\circ)$ the standard deviation of the phase error for pulsed LO is 5.5732° (11.1463°) and 5.8171° (11.6343°) for 50 and 35% duty cycle, respectively. This gives a receiver sensitivity penalty for pulsed LO relative to CW LO at 10^{-9} bit-error-rate of about 0.03 (1.26) and 0.044 (2.15) dB for 50 and 35% duty cycle with $\theta = 80^\circ$.

3. Costas OPLL

Due to the multiplication process in the RF mixer (Fig. 1c), second order noise terms are generated but these terms are negligible and were not included in the analysis. The phase error variance due to laser phase noise is [3]: $\sigma_{l-CW}^2 = 2\pi\Delta\nu\Big/\left(\xi^{CW}\omega_n^{CW}\right)$ for CW LO. For pulsed LO, it can be shown that the phase error variance is

$$\sigma_{l-P}^2 = 2\pi\Delta\nu\Big/\left(\xi^{P}\omega_n^{P}\right) = 2\pi\Delta\nu\Big/\left(\xi^{CW}\omega_n^{CW}\right) = \sigma_{l-CW}^2 \,.$$

The phase error variance due to shot noise for CW LO is $\sigma_{s-CW}^2 = \left(q/RP_s\right)\left(2B_{PLL}^{CW}\right)$ [3]. For pulsed LO, one can show that $\sigma_{s-P}^2 = \left(1/B_l^2\right)\left(q/RP_s\right)\left(2B_{PLL}^{CW}\right) = \left(1/B_l^2\right)\sigma_{s-CW}^2$, where q is the electronic charge and B_l is defined in the previous section. Therefore, the total phase error variance for pulsed LO in terms of those for CW LO is $\sigma_{\phi_e-P}^2 = \sigma_{l-CW}^2 + \left(1/B_l^2\right)\sigma_{s-CW}^2$. Note that the multiplication process produces cross terms that are not present in the balanced loop analysis. The laser phase noise induced phase error variance remains the same as that for CW LO due to the term $F\left\{P_{LO}^{P}(t)\phi_e(t)\right\}H_{LF}(\omega)$ that can be approximated by $P_{LO}^{CW}F\left\{\phi_e(t)\right\}H_{LF}(\omega)$ using Eq. (1). For the shot noise induced phase error, evaluation of the term $\left|F\left\{\sqrt{P_{LO}^{P}(t)}d(t)n_s(t)\right\}\right|^2$ leads to the coefficient $1/B_l^2$. Therefore, the total phase error variance for pulsed LO in terms of phase error variances of CW LO for 50 and 35% duty cycle of the LO pulse is

$$\sigma_{\phi_e-P}^2 = \begin{cases} \sigma_{l-CW}^2 + 0.7253\sigma_{s-CW}^2, & \text{for 50\% duty cycle,} \\ \sigma_{l-CW}^2 + 0.6404\sigma_{s-CW}^2, & \text{for 35\% duty cycle.} \end{cases}$$

In summary, theoretical comparison of phase noise performance of balanced and Costas OPLLs using CW and pulsed LO was conducted. Analytical expressions of phase error variances for pulsed LO in terms of CW LO are presented. For balanced OPLL, the laser phase and shot noise induced phase errors are higher for pulsed LO while the data-induced phase error is lower for pulsed LO. For Costas OPLL, CW and pulsed LO have the same performance except for shot noise induced phase error which is lower for pulsed LO. Phase error variances due to ASE noise for the two OPLLs with pulsed LO have also been derived. Details of the results will be presented.

References

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